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POLARIZATION AND SPIN CORRELATIONS OF TOP QUARKS AT A FUTURE e^+e^- LINEAR COLLIDER

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ABSTRACT

We discuss the polarization and spin correlations of top quarks produced above threshold at a future linear collider, including QCD radiative corrections.

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1 Introduction

Top quark production at a future e^+e^- linear collider provides an excellent possibility to study polarization phenomena of quarks without hadronization ambiguities in a 'clean' environment. Due to its short lifetime, the top quark decays as a quasi-free quark, before hadronization effects can take place. The large width of the top quark thus serves effectively as a cutoff for non-perturbative effects. Information on the polarization and spin correlations of top quarks is therefore not diluted by hadronization effects but transferred to the decay products. This means that the underlying dynamics of both the production and decay process of the heaviest elementary particle known to date can be studied in greater detail, leading to either a confirmation of the Standard Model (SM) predictions or to hints for 'new' physics. For example, the chirality structure of the tWb vertex can be tested with a highly polarized top quark sample [1]. Further, anomalous CP-violating dipole form factors contributing to the $Zt\bar{t}$ and $\gamma t\bar{t}$ vertex would show up as nonzero expectation values of CP-odd spin observables (see, e.g., [2]). Needless to say, the predictions of the SM must be known to high precision in order to establish possible deviations.

The polarization and spin correlations of top quarks can be traced in the angular-energy distributions and momentum correlations of the decay products. Consider for example the decay distribution of charged leptons in semileptonic decays $t \to \ell^+\nu_\ell b$. At leading order within the SM, this distribution reads in the top quark rest frame [3]

$$\frac{d^2\Gamma}{dE_\ell d\cos\theta} = \frac{1}{2}\left(1 + |\mathbf{P}_t|\cos\theta\right)\frac{d\Gamma}{dE_\ell}, \tag{1}$$

where E_ℓ is the energy of the charged lepton and θ is the angle between the direction of ℓ^+ and the polarization $\mathbf{P}_t$ of the top quark sample. A remarkable feature of (1) is the factorization into an energy-dependent and angular-dependent part, which is also respected to a high degree of accuracy by QCD corrections [4]. The direction of flight of the charged lepton in the top quark rest frame is thus a perfect analyser of the top quark polarization. Analogously, angular correlations between ℓ^+ and ℓ^- efficiently probe spin correlations between t and $\bar{t}$. Of course, momenta of other final state particles in semileptonic decays as well as in hadronic top decays can also be used to probe top quark spin effects.

In the remainder of this paper we discuss the polarization and spin correlations in $e^+e^- \to t\bar{t}X$ to order α_s within the SM. Neglecting so-called non-factorizable contributions it is straightforward to combine our results with the known decay distributions of polarized top quarks.

2 Review of leading order results

In this section we write down in a compact form leading order results for the top quark polarization and the spin correlations between t and $\bar{t}$ in the reaction

$$e^+(p_+, \lambda_+) + e^-(p_-, \lambda_-) \to (\gamma^*, Z^*) \to t(k_t) + \bar{t}(k_{\bar{t}}) + X, \tag{2}$$

where λ_- (λ_+) denotes the longitudinal polarization of the electron (positron) beam[1]. Spin effects of top quarks in reaction (2) have been analysed first in ref. [5]. A more recent analysis of spin correlations at leading order can be found in ref. [6], where a so-called 'optimal' spin basis is constructed.

The top quark polarization is defined as two times the expectation value of the top quark spin operator $\mathbf{S}_t$. The operator $\mathbf{S}_t$ acts on the tensor product of the t and $\bar{t}$ spin spaces and is given by $\mathbf{S}_t = \frac{\boldsymbol{\sigma}}{2} \otimes 1\!\!1$, where the first (second) factor in the tensor product refers to the t ($\bar{t}$) spin space. (The spin operator of the top antiquark is defined by $\mathbf{S}_{\bar{t}} = 1\!\!1 \otimes \frac{\boldsymbol{\sigma}}{2}$.) The expectation value is taken with respect to the spin degrees of freedom of the $t\bar{t}$ sample described by a spin density matrix ρ, i.e.

$$\mathbf{P}_t = 2\,\langle \mathbf{S}_t \rangle = 2\frac{\mathrm{Tr}\,[\rho\,\mathbf{S}_t]}{\mathrm{Tr}\,\rho}. \tag{3}$$

For details on the definition and computation of ρ, see [7]. The polarization of the top antiquark $\mathbf{P}_{\bar{t}}$ is defined by replacing $\mathbf{S}_t$ by $\mathbf{S}_{\bar{t}}$ in (3). For top quark pairs produced by CP invariant interactions, $\mathbf{P}_{\bar{t}} = \mathbf{P}_t$. The spin correlations between t and $\bar{t}$ are encoded in the matrix

$$C_{ij} = 4\,\langle S_{t,i} S_{\bar{t},j} \rangle = 4\frac{\mathrm{Tr}\,[\rho\, S_{t,i} S_{\bar{t},j}]}{\mathrm{Tr}\,\rho}. \tag{4}$$

The definitions (3) and (4) imply that $\mathbf{P}_t$ and C_{ij} are independent of the choice of the spin basis. It is convenient to write the results in terms of the

[1] For a right-handed electron (positron), $\lambda_\mp = +1$.

electron and top quark directions $\hat{\mathbf{p}}$ and $\hat{\mathbf{k}}$ defined in the c.m. system, the cosine of the scattering angle $z=\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}$, the scaled top quark mass $r=2m_t/\sqrt{s}$ and the top quark velocity $\beta=\sqrt{1-r^2}$. The electroweak couplings that enter the results are given by

$$\begin{aligned}
g^{VV}_{PC(PV)} &= Q_t^2\, f^{\gamma\gamma}_{PC(PV)} + 2\, g^t_v\, Q_t\, \chi\, f^{\gamma Z}_{PC(PV)} + g^{t\,2}_v\, \chi^2\, f^{ZZ}_{PC(PV)},\\
g^{AA}_{PC(PV)} &= g^{t\,2}_a \chi^2 f^{ZZ}_{PC(PV)},\\
g^{VA}_{PC(PV)} &= -g^t_a\, Q_t\, \chi\, f^{\gamma Z}_{PC(PV)} - g^t_v\, g^t_a\, \chi^2 f^{ZZ}_{PC(PV)},
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
f^{\gamma\gamma}_{PC} &= 1-\lambda_-\lambda_+,\\
f^{\gamma\gamma}_{PV} &= \lambda_- - \lambda_+,\\
f^{ZZ}_{PC} &= (1-\lambda_-\lambda_+)(g^{e2}_v + g^{e2}_a) - 2(\lambda_- - \lambda_+) g^e_v g^e_a,\\
f^{ZZ}_{PV} &= (\lambda_- - \lambda_+)(g^{e\,2}_v + g^{e\,2}_a) - 2\,(1-\lambda_-\lambda_+) g^e_v\, g^e_a,\\
f^{\gamma Z}_{PC} &= -(1-\lambda_-\lambda_+) g^e_v + (\lambda_- - \lambda_+) g^e_a,\\
f^{\gamma Z}_{PV} &= (1-\lambda_-\lambda_+) g^e_a - (\lambda_- - \lambda_+) g^e_v.
\end{aligned} \tag{6}$$

In (5), Q_t denotes the electric charge of the top quark in units of $e=\sqrt{4\pi\alpha}$, and g^f_v, g^f_a are the vector- and the axial-vector couplings of a fermion of type f, i.e. $g^e_v=-\frac{1}{2}+2\sin^2\vartheta_W$, $g^e_a=-\frac{1}{2}$ for an electron, and $g^t_v=\frac{1}{2}-\frac{4}{3}\sin^2\vartheta_W$, $g^t_a=\frac{1}{2}$ for a top quark, with ϑ_W denoting the weak mixing angle. The function χ is given by

$$\chi = \frac{1}{4\sin^2\vartheta_W \cos^2\vartheta_W}\,\frac{s}{s-m_Z^2}, \tag{7}$$

where m_Z stands for the mass of the Z boson.

We further introduce a vector perpendicular to $\mathbf{k}$ in the production plane, $\mathbf{k}^\perp = \hat{\mathbf{p}} - z\hat{\mathbf{k}}$. A simple calculation yields:

$$\mathbf{P}_t = 2\frac{r\left(\beta z g^{VA}_{PC} + g^{VV}_{PV}\right)\mathbf{k}^\perp + \left[\beta(1+z^2) g^{VA}_{PC} + z g^{VV}_{PV} + \beta^2 z g^{AA}_{PV}\right]\hat{\mathbf{k}}}{\left[2-\beta^2(1-z^2)\right] g^{VV}_{PC} + \beta^2(1+z^2) g^{AA}_{PC} + 4\beta z g^{VA}_{PV}}, \tag{8}$$

$$
\begin{aligned}
C_{ij} &= \frac{1}{3}\delta_{ij} + \frac{2}{\left[2-\beta^2(1-z^2)\right] g_{PC}^{VV} + \beta^2(1+z^2) g_{PC}^{AA} + 4\beta z g_{PV}^{VA}} \\
&\times \Bigg[\left(\left[z^2 + \beta^2(1-z^2)\right] g_{PC}^{VV} + \beta^2 z^2 g_{PC}^{AA} + 2\beta z g_{PV}^{VA} \right) \left(\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij} \right) \\
&\quad + (g_{PC}^{VV} - \beta^2 g_{PC}^{AA}) \left(k_i^{\perp} k_j^{\perp} - \frac{1}{3}\delta_{ij}(1-z^2) \right) \\
&\quad + r(z g_{PC}^{VV} + \beta g_{PV}^{VA})(k_i^{\perp}\hat{k}_j + k_j^{\perp}\hat{k}_i) \Bigg]. \qquad (9)
\end{aligned}
$$

In the limit $\beta \to 0$ (threshold) we obtain for the top quark polarization:

$$
\mathbf{P}_t \overset{\beta\to 0}{\longrightarrow} \frac{g_{PV}^{VV}}{g_{PC}^{VV}}\hat{\mathbf{p}} + \beta \left[\left(\frac{g_{PC}^{VA}}{g_{PC}^{VV}} - 2\frac{g_{PV}^{VV} g_{PV}^{VA}}{(g_{PC}^{VV})^2} \right) z\,\hat{\mathbf{p}} + \frac{g_{PC}^{VA}}{g_{PC}^{VV}}\hat{\mathbf{k}} \right] + \mathcal{O}(\beta^2). \qquad (10)
$$

In the leading order parton model calculation, the top quark polarization becomes parallel to the electron beam for $\beta = 0$. For a fully polarized electron beam (and unpolarized positrons), we have $g_{PV}^{VV(VA)} = \pm g_{PC}^{VV(VA)}$ for $\lambda_- = \pm 1$. In that case the top quark polarization along the beam is equal to the electron polarisation, $\mathbf{P_t} \cdot \hat{\mathbf{p}} = \lambda_- = \pm 1$, up to corrections of order β^2.

The spin correlations also have a simple limit:

$$
C_{ij} \overset{\beta\to 0}{\longrightarrow} \hat{p}_i\hat{p}_j + \beta \frac{g_{PV}^{VA}}{g_{PC}^{VV}} (\hat{p}_i\hat{k}_j + \hat{p}_j\hat{k}_i - 2z\hat{p}_i\hat{p}_j) + \mathcal{O}(\beta^2). \qquad (11)
$$

Note that in the threshold region QCD binding effects modify the above parton model results significantly. More precisely, the simple factor β in (10) and (11) gets replaced by a function incorporating the complex dynamics of the $t\bar{t}$ system close to threshold, which is governed by the QCD potential [8].

In the high-energy limit $r = \sqrt{1-\beta^2} \to 0$,

$$
\mathbf{P}_t \overset{r\to 0}{\longrightarrow} 2\,\frac{(1+z^2) g_{PC}^{VA} + z(g_{PV}^{VV} + g_{PV}^{AA})}{(1+z^2)(g_{PC}^{VV} + g_{PC}^{AA}) + 4z g_{PV}^{VA}}\hat{\mathbf{k}} + \mathcal{O}(r), \qquad (12)
$$

i.e. the top quark polarization becomes parallel to its direction of flight.

Finally,

$$
\begin{aligned}
C_{ij} \quad \stackrel{r\to 0}{\longrightarrow} \quad & \frac{1}{3}\delta_{ij} + \frac{2}{(1+z^2)(g_{PC}^{VV}+g_{PC}^{AA})+4g_{PV}^{VA}z} \\
\times \quad & \Big[(g_{PC}^{VV}+z^2 g_{PC}^{AA}+2zg_{PV}^{VA}) \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij} \right) \\
& + (g_{PC}^{VV}-g_{PC}^{AA}) \left(k_i^{\perp}k_j^{\perp} - \frac{1}{3}\delta_{ij}(1-z^2) \right) \Big] + \mathcal{O}(r). \qquad (13)
\end{aligned}
$$

3 QCD corrections at order α_s

The QCD corrections at order α_s to the above results are given by the contributions from one-loop virtual corrections to $e^+e^- \to t\bar{t}$ and from the real gluon emission process $e^+e^- \to t\bar{t}g$. The so-called phase space slicing method is used to isolate the soft gluon singularities. The contribution of hard gluons to $\mathbf{P}_t$ and C_{ij} is computed by numerically integrating all phase space variables of the $t\bar{t}g$ final state except for the top quark scattering angle. Further details of the computation are given in ref. [7]. Results to order α_s for the polarization projected onto $\hat{\mathbf{k}}$ and $\mathbf{k}^{\perp}/|\mathbf{k}^{\perp}|$ can also be found in ref. [9] and ref. [10], respectively.

Absorptive parts of the one-loop amplitude induce as new structures a polarization normal to the event plane [5, 11, 12] as well as new types of spin correlations. We denote these additional structures by an upper index 'abs'. Defining $\mathbf{n} = \hat{\mathbf{p}} \times \hat{\mathbf{k}}$, they read:

$$
\begin{aligned}
\mathbf{P}_t^{\mathrm{abs.}} \quad = \quad & \frac{\alpha_s C_F r \left[(\beta^2-2)g_{PV}^{VA} - \beta z g_{PC}^{VV} \right]}{2\left(g_{PC}^{VV}\left[2-\beta^2(1-z^2)\right] + g_{PC}^{AA}\beta^2(1+z^2) + 4g_{PV}^{VA}\beta z\right)}\,\mathbf{n} \\
\stackrel{\beta\to 0}{\longrightarrow} \quad & = -\frac{\alpha_s C_F}{2}\frac{g_{PV}^{VA}}{g_{PC}^{VV}}\,\mathbf{n} + \mathcal{O}(\beta), \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
C_{ij}^{\mathrm{abs.}} \quad = \quad & \frac{-\alpha_s C_F r}{2\left(g_{PC}^{VV}\left[2-\beta^2(1-z^2)\right] + g_{PC}^{AA}\beta^2(1+z^2) + 4g_{PV}^{VA}\beta z\right)} \\
\times \quad & \left[(\beta g_{PV}^{VV} - (\beta^2-2)zg_{PC}^{VA})(n_i\hat{k}_j + \hat{k}_i n_j) \right. \\
& \left. + 2rg_{PC}^{VA}(n_i k_j^{\perp} + k_i^{\perp} n_j) \right] \\
\stackrel{\beta\to 0}{\longrightarrow} \quad & = -\frac{\alpha_s C_F}{2}\frac{g_{PC}^{VA}}{g_{PC}^{VV}}\,(n_i\hat{p}_j + \hat{p}_i n_j) + \mathcal{O}(\beta), \qquad (15)
\end{aligned}
$$

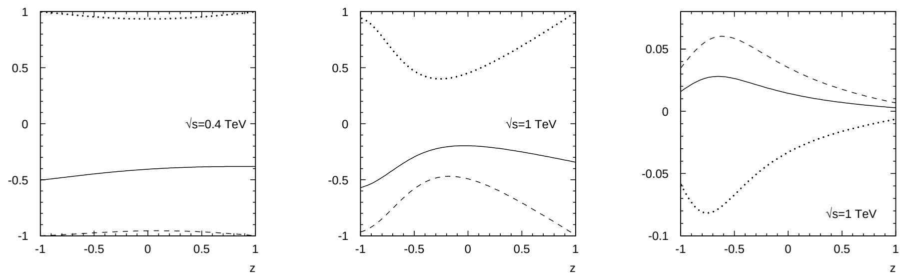

Figure 1: Top quark polarization projected onto the direction of the electron, i.e. $\mathbf{P}_t \cdot \hat{\mathbf{p}}$, as a function of z. The left and middle figure are the results including the order α_s corrections, the right figure shows the value of the QCD correction itself at $\sqrt{s} = 1$ TeV. Input values: $m_t = 175$ GeV, $\alpha_s = 0.1$ (fixed), $\sin^2 \vartheta_W = 0.2236$, and $\lambda_+ = 0$. The solid line is for $\lambda_- = 0$, the dashed line for $\lambda_- = -1$, and the dotted line for $\lambda_- = +1$.

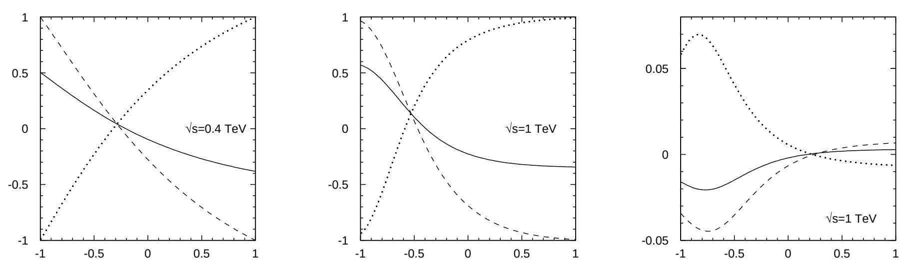

Figure 2: Same as Fig. 1, but for $\mathbf{P}_t \cdot \hat{\mathbf{k}}$.

where $C_F = 4/3$. The threshold behaviour of the other order α_s corrections to the Born results for $\mathbf{P}_t$ and C_{ij} is very simple: First, all these corrections vanish at $\beta = 0$. Second, the QCD corrections of order β can be implemented in the Born formulas (10) and (11) by multiplying the respective order β term with the factor $(1 + \alpha_s C_F/\pi)$.

We now turn to the discussion of numerical results obtained from the exact calculation including the QCD corrections. We consider unpolarized positron beams and the three cases $\lambda_- = 0, \pm 1$. For c.m. energies not too far from the $t\bar{t}$ threshold, the QCD corrections are quite small. (As mentioned before, the parton model results presented here can not be used

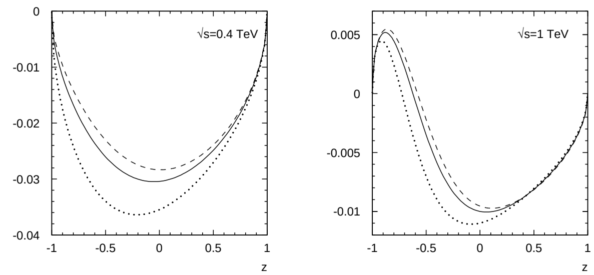

Figure 3: Top quark polarization projected onto $\hat{\mathbf{n}} = \mathbf{n}/|\mathbf{n}|$, i.e. $\mathbf{P}_t \cdot \hat{\mathbf{n}}$, which is zero at Born level. The labelling of the curves is as in Fig. 1.

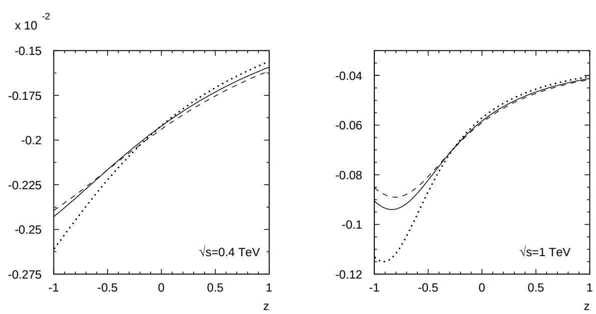

Figure 4: Order α_s correction to the correlation $C_{ij}\delta_{ij} = 4\langle \mathbf{S}_t \cdot \mathbf{S}_{\bar{t}} \rangle$, which is equal to 1 at the Born level. The labelling of the curves is as in Fig. 1.

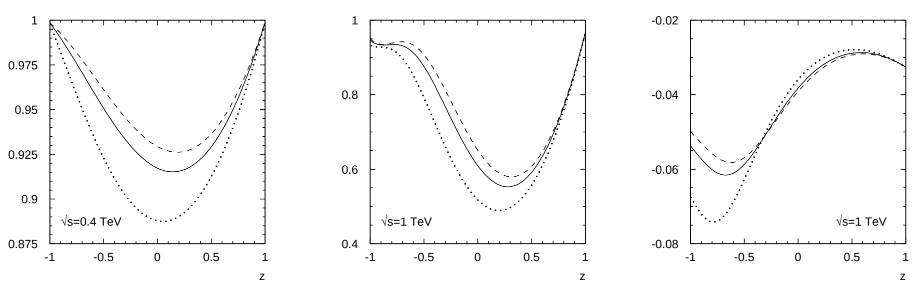

Figure 5: Same as Fig. 1 , but for the correlation $\hat{p}_i C_{ij} \hat{p}_j$.

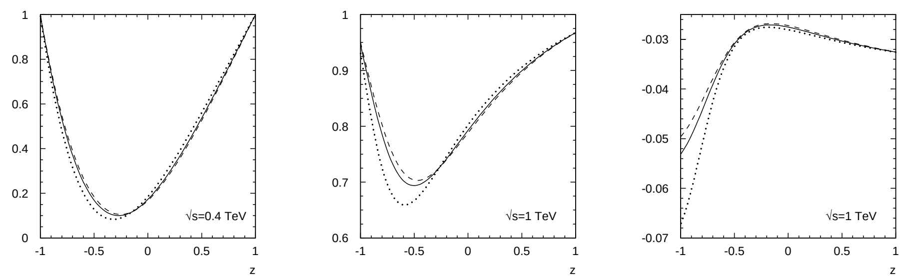

Figure 6: Same as Fig. 1 , but for the correlation $\hat{k}_i C_{ij} \hat{k}_j$.

in the threshold region itself, where the expansion in α_s does not make sense.) For example, at $\sqrt{s} = 0.4$ TeV the corrections are smaller than 0.5% (1%) for the top quark polarization projected onto the electron beam (top quark direction of flight) for all scattering angles. Far above threshold, the QCD correction to the polarization can reach values above 5% for special values of z (see right plots of Figs. 1 and 2). The normal polarization shown in Fig. 3 reaches values of a few percent for not too high c.m. energies.

Since the production of the top quarks proceeds through a single spin-one gauge boson, the correlation $C_{ij}\delta_{ij} = 4\langle \mathbf{S}_t \cdot \mathbf{S}_{\bar{t}} \rangle$ is exactly equal to 1 at the Born level, independent of the scattering angle. Only hard gluon emission leads to a deviation from this result. The QCD correction to this correlation is therefore extremely small at $\sqrt{s} = 0.4$ TeV due to the phase space suppression (see Fig. 4, left). However, at $\sqrt{s} = 1$ TeV, the hard gluon emission leads to a substantial decrease of this correlation, which exceeds 10% for top quarks emitted in the backward direction in the case of right-handed electron beams (Fig. 4, right). Fig. 5 shows the 'beamline' spin correlation $\hat{p}_i C_{ij} \hat{p}_j$. The QCD corrections to this quantity are smaller than 1% at $\sqrt{s} = 0.4$ TeV and of the order of 5% at $\sqrt{s} = 1$ TeV (right plot of Fig. 5). Finally, Fig. 6 depicts our results for the correlation $\hat{k}_i C_{ij} \hat{k}_j$. Note that this correlation is at Born level equal to (-1) times the 'helicity' correlation $P_{\ell\ell} = \hat{k}_{t,i} C_{ij} \hat{k}_{\bar{t},j}$. This special spin correlation, averaged over the scattering angle, was computed analytically to order α_s in ref. [13, 14]. Further results for other c.m. energies and for additional spin observables can be found in ref. [7].

4 Conclusions

At a future linear collider, it will be possible to precisely study the rich phenomenology of top quark spin effects, both at threshold and in the continuum. Theoretical predictions for the top quark polarization and the $t\bar{t}$ spin correlations above threshold are available to order α_s. The QCD corrections are in general small not too far away from threshold, but can reach, for energies around 1 TeV, values of the order of 5% or larger in certain kinematic regions. Their inclusion is mandatory in searches for nonstandard interactions of the top quark.

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